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SOLUTION OF PROBLEMS ON THE ANTI-PLANE DEFORMATION OF ELASTIC BODIES WITH CORNER POINTS BY THE METHOD OF INTEGRAL EQUATIONS*

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The problem of the antiplane deformation of an elastic cylinder with a multiconnected finite or infinite section, bounded by a system of closed curves that can have corner points, is examined. Forces or displacements are given on the whole boundary of the body. The problem is reduced to an integral equation whose kernel has strong stationary singularities at the corner points. Results of an investigation of the solvability of this equation and the smoothness of its solution are presented. A procedure for the numerical solution of the integral equation is described. A space with a prismatic hole of rectangular section or a rigid inclusion subjected to a uniform tangential force at infinity is considered as an example. The generalized stress intensity factors are calculated.

1. Consider the problem of antiplane deformation of an elastic isotropic cylinder whose section D can be multiconnected and bounded by the system $\Gamma = \Gamma_0 \cup \Gamma_1 \cup \ldots \cup \Gamma_m$ of closed contours of bounded rotation without reentry points. In particular, the boundary of the domain D can have corner points, in which the internal angle is not zero or 2π. The contour Γ_0 encloses all the remaining contours Γ_i. There may be no such contour, in which case the domain under consideration is infinite. Forces (Problem 1) or displacements (Problem 2) are given on the whole boundary. It is known /1/ that if solutions with limited energy are sought at the corner points in such problems, the stresses can have power-law singularities.

Let W be the displacement along the cylinder axis. Then the stresses can be expressed in terms of W in the form

$$\tau_x = \sigma_{xz} = G\frac{\partial W}{\partial x}, \quad \tau_y = \sigma_{yz} = G\frac{\partial W}{\partial y}, \quad \tau_n = \sigma_{nz} = G\frac{\partial W}{\partial n}$$

Here z is a coordinate parallel to the cylinder axis, x, y are Cartesian coordinates in the plane of the section D, and G is the shear modulus. For the equilibrium equation to be satisfied, it is necessary that the displacement W satisfy Laplace's equation

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} = 0 \tag{1.1}$$

Problems are considered with the following boundary conditions:

$$\left.\frac{\partial W}{\partial n}\right|_\Gamma = \frac{\tau_n}{G} = \frac{1}{G} T(s) \ \text{(Problem 1)} \tag{1.2}$$

$$W|_\Gamma = U(s) \quad \text{(Problem 2)} \tag{1.3}$$

Here s is the length of the arc of the contour Γ oriented so that the domain D remains on the left when passing in the positive direction, n is the external normal to Γ, the given displacement is $U(s) \in W_r^1(\Gamma)$, and the given force is $T(s) \in L_r(\Gamma)$, $1 < r < \infty$.

We shall seek the solution of these problems in the class of functions $W \in C^2(D) \cap C(\bar{D})$ and bounded at infinity if the domain is infinite.

The solution of Problem 1 will be sought in the form

$$W^{(1)}(x_1, y_1) = \operatorname{Im}\left\{\int_{z_0}^{z_1} \Phi(z)\,dz\right\} + C \tag{1.4}$$

The solution of Problem 2 will be sought in the form

$$W^{(2)}(x_1, y_1) = \operatorname{Re}\left\{\int_{z_0}^{z_1} [\Phi(z) + \Sigma(z)]\,dz\right\} + A_0, \quad \Sigma(z) = \sum_{i=1}^{m} \frac{A_i}{z - a_i} \tag{1.5}$$

Here $z_1 = x_1 + iy_1$ is a complex coordinate of a point in D, z_0 is an arbitrary fixed point in D, the function $\Phi(z)$ is analytic, C is an arbitrary real constant, a_i are arbitrary points

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within the contours Γ_i, and A_i are real constants for whose determination a method will be given below.

The stresses are expressed in terms of the function Φ as follows:

$$\tau_x^{(1)} = G \operatorname{Im} \Phi(z), \quad \tau_y^{(1)} = G \operatorname{Re} \Phi(z)$$
$$\tau_x^{(2)} = G \operatorname{Re}[\Phi(z) + \Sigma(z)], \quad \tau_y^{(2)} = -G \operatorname{Im}[\Phi(z) + \Sigma(z)]$$

We shall seek the function $\Phi(z)$ in (1.4) and (1.5) in the form of a Cauchy-type integral with real density $\mu(s) \in L_p(\Gamma)$, $1 < p < \infty$

$$\Phi(z) = \frac{1}{\pi i} \int_\Gamma \frac{\mu(s)\, ds}{t - z} \tag{1.6}$$

where $t = t(s)$ is the complex coordinate of a point of the contour, and $k = dt/ds$. Functions of the arguments s and t are not distinguished later, i.e., $g(t) = g(t(s)) = g(s)$. The representation (1.4) and (1.6) is ordinarily the potential of a simple layer, while the representation (1.5) and (1.6) can be expressed in terms of the angular potential used in /2/ for problems in simply-connected domains with smooth boundaries.

After substituting (1.4) and (1.6) into (1.2), and (1.5) and (1.6) into condition (1.3) differentiated with respect to s, we obtain an integral equation in $\mu(s)$ for both problems

$$(I - K)\mu = f, \quad (K\mu)(s_0) = -\frac{1}{\pi} \int_\Gamma \mu(s) \operatorname{Im}\left[\frac{k(s_0)}{t - t_0}\right] ds, \quad (t = t(s_0)) \tag{1.7}$$

$$f(t) = \begin{cases} -T(s)/G & \text{Problem 1} \\ \dfrac{dU(s)}{ds} - \operatorname{Re} \displaystyle\sum_{i=1}^{m} \frac{k(s)\, A_i}{t(s) - a_i} & \text{Problem 2} \end{cases}$$

Representation (1.6) allows of integrable power-law singularities of the stresses if $1 < p < \infty$ and p is sufficiently small. We hence seek the solution of (1.7) $\mu \in L_p$ with a possibly smaller $p > 1$.

Equation (1.7) has been obtained in /3/, Sec.140 for smooth contours. If the contour Γ is a piecewise-Liapunov contour with Hölder index in the Liapunov condition equal to one, then the kernel of (1.6) is bounded everywhere with the exception of corner points where it has a first-order singularity as s and s_0 tend to the corner point from different sides. Such equations were examined in /4, 5/, where formulas were obtained for their index. On the basis of these results, it can be shown that (1.7) in the spaces L_p, $1 < p < p_0$, is a generalized Fredholm equation

$$p_0 = 1 + (\max_j |1 - \omega_j/\pi|)^{-1} \tag{1.8}$$

The maximum in (1.8) is taken over all corner points, while ω_j is the magnitude of the internal angle at these points. It is seen from (1.8) that $p_0 > 2$ in the cases $\omega_j \neq 0$ and $\omega_j \neq 2\pi$. under consideration.

It can be proved that if the contour Γ_0 exists, i.e., the domain does not include infinity, then the homogeneous equation (1.7) has a unique non-trivial solution which nevertheless generates only a zero function $\Phi(z)$ in D according to (1.6), meaning the zeroth stress field in Problem 1 and the stress field generated just by the addition of $\Sigma(z)$ in Problem 2. For the inhomogeneous equation (1.7) to be solvable in the case of a finite domain D, it is necessary that

$$\int_\Gamma f(s)\, ds = 0 \tag{1.9}$$

i.e., that the principal vector of the forces applied to the body be zero in Problem 1, or the displacement $U(s)$ applied to the body boundary be continuous in Problem 2. If there is no contour Γ_0, i.e., the domain D includes the infinitely distant point, then (1.7) is uniquely and absolutely solvable.

To determine A_i in the solution of Problem 2, we impose the condition $W^{(2)}(x(\xi_i), y(\xi_i)) = U(\xi_i), (i = 0, \ldots, m)$, where ξ_i are arbitrary fixed points of the appropriate contours Γ_i, and $W^{(2)}$ is given by (1.5). In the case of no Γ_0 we impose the condition $A_1 + A_2 + \ldots + A_m = 0$, which ensures the boundedness of W at infinity. In both cases, for $m + 1$ unknowns we obtain $m + 1$ linear algebraic equations.

In fact, let Φ_0 be the solution of (1.7) with the right side dU/ds, and let Φ_i $(i = 1, \ldots, m)$ be the solutions of (1.7) with the right sides $\operatorname{Re}[k(s)/(t(s) - a_i)]$. Then the solution of (1.7) with the given right side

$$f = \frac{dU}{ds} - \mathrm{Re} \sum_{i=1}^{m} \frac{k(s) A_i}{t(s) - a_i}$$

equals

$$\Phi = \Phi_0 - \sum_{i=1}^{m} A_i \Phi_i$$

We obtain the system

$$\sum_{j=1}^{m} A_j \left\{ \ln \left| \frac{\xi_i - a_j}{z_0 - a_j} \right| - \mathrm{Re} \left[\int_{z_0}^{\xi_i} \Phi_j(z)\, dz \right] \right\} + A_0 = U(\xi_i) - \mathrm{Re} \left[\int_{z_0}^{\xi_i} \Phi_0(z)\, dz \right], \quad i = 0, \ldots, m$$

If there is no Γ_0, then the equation $A_1 + A_2 + \ldots + A_m = 0$ is added to this system. The system determinant differs from zero because otherwise a solution would be found for the Dirichlet problem with zero boundary conditions, that is different from zero. This reasoning is similar to that presented earlier (/6/.Sec/63). For a numerical solution it is necessary to determine Φ_i $(i = 0, \ldots, m)$ and then A_i.

If the domain D is simply-connected, then there is no need to determine A_i, Φ_i. When it is bounded, the single constant A_0 can be taken in the form $A_0 = U(\xi_0)$ while simultaneously letting the point z_0 tend to ξ_0. When the domain D is not bounded, we set $A_1 = 0$ for the solution to be bounded at infinity and proceed with the remining constant A_0 as in the case of the bounded domain. In both cases $\Phi = \Phi_0$.

The fact that (1.7) lies in the spectrum for the finite domain D can complicate its numerical solution. In this case we turn from (1.7) to the equation

$$(I - K - K_1)\mu = f_1, \quad (K_1\mu)(s_0) = \varepsilon \int_\Gamma \mu(s)\, ds \tag{1.10}$$

where ε is an arbitrary constant different from zero. Using the generalized Schmidt lemma /7/ (see /8/ also), it can be shown that (1.10) will be absolutely and uniquely solvable in L_p, $1 < p < p_0$, and if condition (1.9) for the solvability of (1.7) is satisfied the solution of (1.10) agrees with one of the solutions of (1.7) such that

$$\int_\Gamma \mu(s)\, ds = 0$$

Since, as has been mentioned above, the solution of the homogeneous equation (1.7) does not generate a non-zero function $\Phi(z)$ in (1.6), it does not matter which of the solutions of (1.7) will be the solution of (1.10).

The smoothness of the solutions of (1.7) as a function of the smoothness of the boundary and the boundary conditions is investigated successfully by methods similar to that described earlier /9/. In particular, if s^* is a point of smoothness of the curve Γ, we differentiate the angle φ between the tangent to the curve Γ and the x axis continuously with respect to s in the neighbourhood of this point, and the function $f(s)$ is continuous (i.e., the force or derivative of the given displacement is given continuously), then the solution μ will also be continuous in this neighbourhood. If the function $f(s)$ has a discontinuty of the first kind of magnitude Δf at the point where the curve Γ is sufficiently smooth, then the solution of the integral equation also has a discontinuity of the first kind of this point, and its magnitude is $\Delta\mu = \Delta f$. If s^* is a corner point of the curve Γ, the angle φ in the left and right neighbourhoods of this point is a continuously differentiable function of s, and the function $f(s)$ is sufficiently smooth, for instance, it belongs to the space $W_2^1(\Gamma)$ in the left and right neighbourhoods of s^*, then μ has the following form in these neighbourhoods:

$$\mu^+(s) = \mu_0^+ + A\,|s - s^*|^{-\delta}, \quad \mu^+(s) = \mu_0^+ + A\,|s - s^*|^{-\delta}\,\mathrm{sign}\,(\pi - \omega), \quad \delta = 1 - (1 + |\omega - \pi|/\pi)^{-1} \tag{1.11}$$

Here μ^+ and μ^- are the values of μ in the left and right neighbourhoods of the point s^*, respectively, $\mu_0^\pm$ are sufficiently smooth, say, Hölder functions in the left and right neighbourhoods of s^* that equal zero at the point s^*, ω is an internal angle at the point s^*, and A is a certain unknown constant.

As is seen from (1.11), the density μ in the neighbourhood of corner points has power-law singularities, where it is even singular at those points in which the internal angle ω is less than π and the stresses have no singularities. Using (1.6) and the results obtained in /6/ about the behaviour of a Cauchy-type integral in the neighbourhood of a corner point, we obtain from (1.11) that in the neighbourhood of the corner point s^*, if the part of Γ belonging to s^* is sufficiently smooth, the stresses in a local polar coordinate system r, θ with origin at the point s^*, will have the following form for $\omega > \pi$:

$$\begin{aligned}
&\tau_r^{(1)} = (K^\circ/\sqrt{2\pi})\, r^{-\delta} \sin[\theta(1-\delta)] + \tau_{r0} + \tau_{r1} \\
&\tau_\theta^{(1)} = (K^\circ \sqrt{2\pi})\, r^{-\delta} \cos[\theta(1-\delta)] + \tau_{\theta 0} + \tau_{\theta 1} \\
&K^\circ = -2AG \operatorname{ctg}(\delta\pi)\sqrt{2\pi} \\
&\tau_{r0} = [f^- \cos(\theta + \omega/2) + f^+ \cos(\theta - \omega/2)]/\sin\omega \\
&\tau_{\theta 0} = -G\,[f^- \sin(\theta + \omega/2) + f^+ \sin(\theta - \omega/2)]\,/\,\sin\omega
\end{aligned} \tag{1.12}$$

Here τ_{r1}, $\tau_{\theta 1}$ are continuous functions that tend to zero as $z \to t^*$, f^- and f^+ are the left and right limit values of $f(t)$ as $t \to t^*$, θ is an angle measured anticlockwise from the inner bisectrix at the corner point t^*. As ω tends to 2π, i.e., as $\delta \to {}^1/_2$, formula (1.12) reduces to the usual formula for a longitudinal shear crack. For $\omega < \pi$ we should set $K^\circ = 0$ in (1.12).

It is seen from the representation (1.12) that the second bounded term of the asymptotic form of the stress in the neighbourhood of a corner point is expressed explicitly in terms of the boundary values around the corner point, and it can be found without solving the initial boundary value problem. In particular, it can be shown that the bounded terms τ_{r0} and $\tau_{\theta 0}$ vanish in the problem of pure torsion of a rod in which the tangential forces on the side surfaces are zero. If $\omega > \pi$ then the stresses are represented as the sum of a singular term and a term that is zero at the corner point, and if $\omega < \pi$ then the stresses tend to zero as one approaches the corner point.

Let s^* be a point of smoothness of Γ, at which the function f has a discontinuity of the first kind (i.e., the force $T(s)$ or the derivative of the displacement $dU(s)/ds$) is given) equal to Δf. Then it follows from /6/ that the stresses have the form

$$\tau_r^{(1)} = -G\frac{\Delta f}{\pi}\sin(\theta)\ln r + \tau_{r3}, \quad \tau_\theta^{(1)} = -G\frac{\Delta f}{\pi}\cos(\theta)\ln r + \tau_{\theta 3} \tag{1.13}$$

where τ_{r3}, $\tau_{\theta 3}$ are bounded functions.

For Problem 2 we have $2\tau_r^{(2)} = \tau_\theta^{(1)}$, $\tau_\theta^{(2)} = -\tau_r^{(1)}$, where (1.12) and (1.13) presented above must be taken as $\tau_r^{(1)}$, $\tau_\theta^{(1)}$.

2. Let us examine the problem of the numerical solution of (1.7) and (1.10). The behaviour of the solutions of these equations in the neighbourhood of singular points, to which the corner points of Γ and the points of the discontinuity f in smooth parts of Γ belong, is known. Hence, the desired function μ can be represented in the form $\mu(s) = w(s)\mu_1(s)$, where $\mu_1(s)$ is an unknown bounded function, and $w(s)$ is a known singular weighting function equal to $w(s) = (s - s_1^*)^{-\delta_1}(s_2^* - s)^{-\delta_2}$ on each curvilinear segment between two singular points s_1^* and s_2^*, where δ_1, δ_2 are the degrees of singularity of μ, respectively, at the beginning s_1^* and end s_2^* of the segment. If one of these ends is the point of smoothness of Γ, then $\delta = 0$ there, while δ is determined by (1.11) at the rest of the points.

We turn from (1.7) to the equation

$$\mu_1(s_0) + \frac{1}{\pi w(s_0)}\int_\Gamma \mu_1(s)\, w(s) \operatorname{Im}\left[\frac{k(s_0)}{t - t_0}\right] ds = \frac{f(s_0)}{w(s_0)} \tag{2.1}$$

We reduce this equation to a system of linear algebraic equations by the method of collocation, and we estimate the integral in (2.1) approximately for this by using a quadrature formula of the trapezoid method that takes account of the presence of the singular weight, and we use (2.1) at the angular points of the quadrature formula. We solve the system obtained by the Gauss method of elimination. After finding the values of $\mu_1(s)$ at the nodes, we obtain its values at the angular points by extrapolation over several of the nearest points by using a Lagrange polynomial.

The point distribution over the segment

$$s_j = s_1^* + (s_2^* - s_1^*)\, j/(N+1)$$

The distribution

$$s_j = s_1^* + (s_2^* - s_1^*)(2j-1)/(2N)$$

or the zeros of the Chebyshev polynomials

$$s_j = s_2^* - (s_2^* - s_1^*)\cos^2[(2j-1)\pi/(4N)], \quad j = 1, \ldots, N$$

were selected as nodal points.

The greatest accuracy is achieved when using the Chebyshev points since they are considerably more condensed as one approaches the singular points.

3. As an illustration, consider the problem of antiplane deformation of a space with an infinite prismatic hole of rectangular section or a rigid inclusion. A uniform shear stress τ is applied to the space at infinity, which is parallel to one of the sides of the rectangle in the plane of the section. By superposition we reduce the problem to a problem in a space with forces or displacements applied on the sides of the hole and zero stresses at infinity.

The degree of singularity of the stress at right angles is $\delta = 1/3$.

We introduce two dimensionless intensity coefficients

$$K_* = -K^\circ/[2\operatorname{ctg}(\delta\pi)\sqrt{2\pi}\,\tau a^\delta] = A/(\tau a^\delta),\quad K_{**} = K_*(a/b)^\delta$$

Here a is half the length of a side of the rectangle perpendicular to the direction of the stresses acting at infinity in the plane of the section, and b is half the length of the other side.

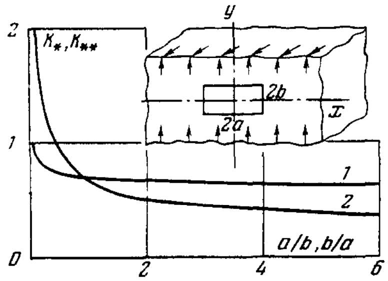

Fig.1

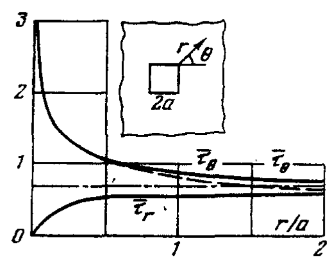

Fig.2

The dependence of the dimensionless stress intensity coefficients on the relative dimensions of the rectangle is represented in Fig.1. The ratio b/a is laid off on the horizontal axis for the hole, and a/b for the inclusion. Curve *1* is the dependence of K_* on the ratio b/a for the hole and the dependence of K_{**} on the ratio a/b for the inclusion. Curve *2* is the dependence of K_{**} on the ratio b/a for the hole and the dependence of K_* on the ratio a/b for the inclusion.

These graphs show the following. When the problem with a hole is considered, the stress intensity coefficient K° drops for fixed a as b increases and emerges on the asymptote corresponding to the intensity coefficient for a slit in the form of a half-strip with the slit axis parallel to the direction of the shear stresses in the plane of the section (the y axis), in which case $K_* = 0.66$. If the dimension b decreases for fixed a, the generalized intensity coefficient increases without limit since in the limit we obtain a crack along the x axis, in which the degree of singularity is, as is known, $1/2 > \delta$ and the intensity coefficient for this singularity is different from zero. When b is fixed, and a increases we obtain a slit in the form of a half-strip along the x axis in the limit, and the intensity coefficient tends to infinity, as might have been expected. For fixed b and a tending to zero, we obtain, in the limit, a zero intensity coefficient since in this case we have a crack parallel to the y axis at which the intensity coefficient equals zero; hence the intensity coefficient for a rectangular hole drops monotonically to zero. Analogous reasoning can be used for the inclusion also.

In Fig.2 we show the distribution of the stresses $\bar\tau_r = \tau_r/\tau$ and $\bar\tau_\theta = \tau_\theta/\tau$ in the local polar coordinate system on the continuation of the diagonal of a square in the problem of a space with a square hole shifted to infinity by uniform forces τ. The distance is measured from the vertex of the square. Also shown in Fig.2 is the distribution of the stress $\bar\tau_\theta^* = \tau_\theta^*/\tau$ given just by the first singular term in the asymptotic form (1.12). In the case under consideration, the second constant term of the asymptotic form is zero and the graph given for $\bar\tau_\theta^*$ actually represents a two-term asymptotic form of the stress $\bar\tau_\theta$. It is seen from the graphs presented that the two-term asymptotic agrees with the true stresses $\bar\tau_\theta$ at fairly short distances from the vertex of the square and approaches $\bar\tau_\theta$ with 10% accuracy at distances less than a. On approaching the vertex of the square, $\bar\tau_r$ tends to zero, as it should do because of (1.12). At fairly large distances $\tau_\theta = \tau_r = \tau/\sqrt{2}$, which corresponds to shear of a space without a hole, while the dash-dot line in Fig.2 represents this asymptote. As is seen from the graphs, at distances greater than 2a, τ_r and τ_θ differ from the limit value by not more than 10%.

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METHODS OF SOLVING SPATIAL PROBLEMS OF THE MECHANICS OF A DEFORMABLE SOLID IN TERMS OF STRESSES*

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The formulation in /1/ of a quasistatic problem of the mechanics of a deformable solid in terms of stresses is discussed, including also the variational formulation, which consists of solving six equations in six symmetric stress tensor components when six boundary conditions are satisfied. Methods of successive approximation are proposed for solving this problem and theorems on the convergence of these methods, including a "rapidly converging" method, whose rate of convergence is substantially higher than a geometric progression, are proved.

Utilization of the Lagrange and Castigliano variational principles in the numerical solution of boundary value problems of the mechanics of a deformable solid enables an a priori stable difference scheme /1/ to be compiled as well as an effective means for solving it. The disadvantages of applying each of these variational principles are well known. Thus, when using the Lagrangian, the desired quantities are displacements, and a numerical differentiation procedure that considerably reduces the accuracy of the solution obtained must be used to determine the state of stress. When using the Castiglianian, the problem is to seek the conditional extremum (in the class of tensor functions satisfying the equilibrium equations and the static boundary conditions), which often turns out to be difficult.

A new variational principle, based on solving the mechanics problem of a deformable solid in terms of stresses /1/ is considered below, and methods of solving the quasistatic problem of physically non-linear mechanics of a deformable solid are described.

1. Consider a physically non-linear medium in which the relation between the strain tensor components ε and the stress tensor components σ is given in the operator form

$$\varepsilon_{ij} = G_{ij}(\sigma) \tag{1.1}$$

On the boundary of a body Σ occupying a volume V let a force vector be given and let the following equilibrium conditions be satisfied:

$$\sigma_{ij}n_j\,|_\Sigma = S_i^\circ, \quad q_i\,|_\Sigma = -X_i\,|_\Sigma \tag{1.2}$$

(X_i are components of the volume force vector).

The quasistatic problem of the mechanics of a deformable body in terms of stresses (Problem B /1/) is to solve six equations in six unknown stress tensor components

$$E_{ijk,k} + Y_{ij} = 0 \tag{1.3}$$

while satisfying boundary conditions (1.2). Here

$$\begin{aligned} E_{ijk} \equiv\ & \varepsilon_{ij,k} + \delta_{ki}(^1/_2\,\varepsilon_{mm,j} - \varepsilon_{mj,m}) + \delta_{kj}(^1/_2\,\varepsilon_{mm,i} - \varepsilon_{mi,m}) + \\ & \xi_{ij}(\varepsilon_{mk,m} - \varepsilon_{mm,k}) + R_i(q) + R_j(q) - \xi_{ij}R_k(q) \end{aligned} \tag{1.4}$$

where their expressions in terms of the stresses (1.1) is substituted in place of the strain ε, ξ is a certain arbitrary symmetric constant tensor, and R is a certain linear vector

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